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Diploma Programme

Mathematics: applications and interpretation HL formula booklet

For use during the course and in the examinations
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Two decorative blue curves are present on the page. One is a thick, dark blue curve that starts on the left, rises to a peak, and then descends towards the right. The other is a thin, light blue curve that starts high on the left, descends to a minimum, and then rises towards the right.

HIGHER LEVEL

Contents

Topic 1: Number and algebra – HL	2
Topic 2: Functions – HL	3
Topic 3: Geometry and trigonometry – HL	4
Topic 4: Statistics and probability – HL	8
Topic 5: Calculus – HL	10

Topic 1: Number and algebra – HL

1.2	<p>The nth term of an arithmetic sequence</p> <p>The sum of n terms of an arithmetic sequence</p>	$u_n = u_1 + (n - 1)d$ $S_n = \frac{n}{2}(2u_1 + (n - 1)d); S_n = \frac{n}{2}(u_1 + u_n)$
1.3	<p>The nth term of a geometric sequence</p> <p>The sum of n terms of a finite geometric sequence</p>	$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$
1.11	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r}, r < 1$
1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest</p>
1.5	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$
1.9	Laws of logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ <p>for $a, x, y > 0$</p>
1.6	Percentage error	$\mathcal{E} = \left \frac{v_A - v_E}{v_E} \right \times 100\%$ <p>where v_E is the exact value and v_A is the approximate value of v</p>
1.12	<p>Complex numbers</p> <p>Discriminant</p>	$z = a + bi$ $\Delta = b^2 - 4ac$
1.13	Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = r e^{i\theta} = r \operatorname{cis} \theta$

1.14	Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = A = ad - bc$
	Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, ad \neq bc$
1.15	Power formula for a matrix	$M^n = PD^nP^{-1}$, where P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues

Topic 2: Functions – HL

Prior learning – HL

Solutions of a quadratic equation	The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
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2.1	Equations of a straight line	$y = mx + c; ax + by + d = 0; y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.5	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
2.9	Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}, L, k, C > 0$

Topic 3: Geometry and trigonometry – HL

Prior learning – HL

Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of prism	$V = Ah$, where A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi r h$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

3.1	Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$
	Volume of a right-pyramid	$V = \frac{1}{3}Ah$, where A is the area of the base, h is the height

	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$, where r is the radius, h is the height
	Area of the curved surface of a cone	$A = \pi r l$, where r is the radius, l is the slant height
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
	Surface area of a sphere	$A = 4\pi r^2$, where r is the radius
3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab \sin C$
3.4	Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle measured in degrees, r is the radius
	Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$, where θ is the angle measured in degrees, r is the radius
3.7	Length of an arc	$l = r\theta$, where r is the radius, θ is the angle measured in radians
	Area of a sector	$A = \frac{1}{2}r^2\theta$, where r is the radius, θ is the angle measured in radians
3.8	Identities	$\cos^2 \theta + \sin^2 \theta = 1$ $\tan \theta = \frac{\sin \theta}{\cos \theta}$

3.9	Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, reflection in the line $y = (\tan \theta)x$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, horizontal stretch / stretch parallel to x -axis with a scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$, vertical stretch / stretch parallel to y -axis with a scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, enlargement, with a scale factor of k , centre $(0, 0)$ $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$, anticlockwise/counter-clockwise rotation of angle θ about the origin ($\theta > 0$) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, clockwise rotation of angle θ about the origin ($\theta > 0$)
3.10	Magnitude of a vector	$ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}$, where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
3.11	<p>Vector equation of a line</p> <p>Parametric form of the equation of a line</p>	$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ $x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$

Topic 4: Statistics and probability – HL

4.2	Interquartile range	$IQR = Q_3 - Q_1$
4.3	Mean, \bar{x} , of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$, where $n = \sum_{i=1}^k f_i$
4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A) P(B)$
4.7	Expected value of a discrete random variable X	$E(X) = \sum_{i=1}^k x_i P(X = x_i)$
4.8	Binomial distribution $X \sim B(n, p)$	
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1 - p)$

<p>4.14</p>	<p>Linear transformation of a single random variable</p> <p>Linear combinations of n independent random variables, X_1, X_2, \dots, X_n</p> <p>Sample statistics</p> <p>Unbiased estimate of population variance s_{n-1}^2</p>	$E(aX + b) = aE(X) + b$ $\text{Var}(aX + b) = a^2 \text{Var}(X)$ $E(a_1X_1 \pm a_2X_2 \pm \dots \pm a_nX_n) = a_1E(X_1) \pm a_2E(X_2) \pm \dots \pm a_nE(X_n)$ $\text{Var}(a_1X_1 \pm a_2X_2 \pm \dots \pm a_nX_n) = a_1^2 \text{Var}(X_1) + a_2^2 \text{Var}(X_2) + \dots + a_n^2 \text{Var}(X_n)$ $s_{n-1}^2 = \frac{n}{n-1} s_n^2$
<p>4.17</p>	<p>Poisson distribution $X \sim \text{Po}(m)$</p> <p>Mean</p> <p>Variance</p>	$E(X) = m$ $\text{Var}(X) = m$
<p>4.19</p>	<p>Transition matrices</p>	$\mathbf{T}^n \mathbf{s}_0 = \mathbf{s}_n, \text{ where } \mathbf{s}_0 \text{ is the initial state}$

Topic 5: Calculus – HL

5.3	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.9	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	
5.5	Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
5.8	The trapezoidal rule	$\int_a^b y dx \approx \frac{1}{2} h ((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})),$ <p>where $h = \frac{b-a}{n}$</p>

5.11	Standard integrals	$\int \frac{1}{x} dx = \ln x + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int \frac{1}{\cos^2 x} dx = \tan x + C$ $\int e^x dx = e^x + C$
5.12	<p>Area of region enclosed by a curve and x or y-axes</p> <p>Volume of revolution about x or y-axes</p>	$A = \int_a^b y dx \text{ or } A = \int_a^b x dy$ $V = \int_a^b \pi y^2 dx \text{ or } V = \int_a^b \pi x^2 dy$
5.13	<p>Acceleration</p> <p>Distance travelled from t_1 to t_2</p> <p>Displacement from t_1 to t_2</p>	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$ $\text{distance} = \int_{t_1}^{t_2} v(t) dt$ $\text{displacement} = \int_{t_1}^{t_2} v(t) dt$
5.16	<p>Euler's method</p> <p>Euler's method for coupled systems</p>	$y_{n+1} = y_n + h \times f(x_n, y_n); x_{n+1} = x_n + h, \text{ where } h \text{ is a constant (step length)}$ $x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ <p>where h is a constant (step length)</p>
5.17	Exact solution for coupled linear differential equations	$\mathbf{x} = Ae^{\lambda_1 t} \mathbf{p}_1 + Be^{\lambda_2 t} \mathbf{p}_2$