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# Mathematics: analysis and approaches SL formula booklet

For use during the course and in the examinations  
First examinations 2021

Version 1.0

# STANDARD LEVEL

Two decorative blue curves are present on the page. One is a thick, dark blue curve that starts on the left, rises to a peak, and then descends towards the right. The other is a thin, light blue curve that starts high on the left, descends to a minimum, and then rises towards the right.



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## Topic 1: Number and algebra – SL

<b>1.2</b>	<p>The <math>n</math>th term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n - 1)d$ $S_n = \frac{n}{2}(2u_1 + (n - 1)d); S_n = \frac{n}{2}(u_1 + u_n)$
<b>1.3</b>	<p>The <math>n</math>th term of a geometric sequence</p> <p>The sum of <math>n</math> terms of a finite geometric sequence</p>	$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$
<b>1.8</b>	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r},  r  < 1$
<b>1.4</b>	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where <math>FV</math> is the future value,  <math>PV</math> is the present value, <math>n</math> is the number of years,  <math>k</math> is the number of compounding periods per year,  <math>r\%</math> is the nominal annual rate of interest</p>
<b>1.5</b>	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$
<b>1.7</b>	<p>Exponents and logarithms</p> <p>Exponential and logarithmic functions</p>	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$ $a^x = e^{x \ln a}; \log_a a^x = x = a^{\log_a x} \text{ where } a, x > 0, a \neq 1$
<b>1.9</b>	Binomial theorem $n \in \mathbb{N}$	$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$ ${}^n C_r = \frac{n!}{r!(n-r)!}$

## Topic 2: Functions – SL

<b>2.1</b>	Equations of a straight line	$y = mx + c; ax + by + d = 0; y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
<b>2.6</b>	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
<b>2.7</b>	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$

## Topic 3: Geometry and trigonometry – SL

### Prior learning – SL

Area of a parallelogram	$A = bh$ , where $b$ is the base, $h$ is the height
Area of a triangle	$A = \frac{1}{2}(bh)$ , where $b$ is the base, $h$ is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$ , where $a$ and $b$ are the parallel sides, $h$ is the height
Area of a circle	$A = \pi r^2$ , where $r$ is the radius
Circumference of a circle	$C = 2\pi r$ , where $r$ is the radius
Volume of a cuboid	$V = lwh$ , where $l$ is the length, $w$ is the width, $h$ is the height
Volume of a cylinder	$V = \pi r^2 h$ , where $r$ is the radius, $h$ is the height
Volume of a prism	$V = Ah$ , where $A$ is the area of cross-section, $h$ is the height
Area of the curved surface of a cylinder	$A = 2\pi r h$ , where $r$ is the radius, $h$ is the height
Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

<b>3.1</b>	Distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$

	Volume of a right-pyramid	$V = \frac{1}{3}Ah$ , where $A$ is the area of the base, $h$ is the height
	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$ , where $r$ is the radius, $h$ is the height
	Area of the curved surface of a cone	$A = \pi r l$ , where $r$ is the radius, $l$ is the slant height
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$ , where $r$ is the radius
	Surface area of a sphere	$A = 4\pi r^2$ , where $r$ is the radius
<b>3.2</b>	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$ ; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab \sin C$
<b>3.4</b>	Length of an arc	$l = r\theta$ , where $r$ is the radius, $\theta$ is the angle measured in radians
	Area of a sector	$A = \frac{1}{2}r^2\theta$ , where $r$ is the radius, $\theta$ is the angle measured in radians
<b>3.5</b>	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
<b>3.6</b>	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

## Topic 4: Statistics and probability – SL

<b>4.2</b>	Interquartile range	$IQR = Q_3 - Q_1$
<b>4.3</b>	Mean, $\bar{x}$ , of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$ , where $n = \sum_{i=1}^k f_i$
<b>4.5</b>	Probability of an event $A$	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
<b>4.6</b>	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)P(B)$
<b>4.7</b>	Expected value of a discrete random variable $X$	$E(X) = \sum_{i=1}^k x_i P(X = x_i)$
<b>4.8</b>	Binomial distribution $X \sim B(n, p)$	
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1-p)$
<b>4.12</b>	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

## Topic 5: Calculus – SL

<b>5.3</b>	Derivative of $x^n$	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
<b>5.6</b>	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of $e^x$	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u)$ , where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
<b>5.9</b>	Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
	Distance travelled from $t_1$ to $t_2$	distance = $\int_{t_1}^{t_2}  v(t)  dt$
	Displacement from $t_1$ to $t_2$	displacement = $\int_{t_1}^{t_2} v(t) dt$
<b>5.5</b>	Integral of $x^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
	Area between a curve $y = f(x)$ and the $x$ -axis, where $f(x) > 0$	$A = \int_a^b y dx$

<b>5.10</b>	Standard integrals	$\int \frac{1}{x} dx = \ln x  + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
<b>5.11</b>	Area of region enclosed by a curve and $x$ -axis	$A = \int_a^b  y  dx$